Unified Analysis of the Metal Vapour Arc

G. Ecker

Institut für Theoretische Physik, Ruhr-Universität Bochum

For given values of the total current density j and the cathode spot surface temperature T a unified and consistent calculation of the cathode drop, U_c, the electron temperature in the ionization region T_-- the electron emission current density j_e, the ion current density j_+, and the extension of the space charge region l_{sp} are presented.

We find that the counter diffusion of plasma electrons into the space charge region plays a decisive role. It causes an effective space charge region extension l_{sp} of a few plasma electron Debey lengths which in general is much less than the ion mean-free-path commonly used. Without the effect of the counter diffusing electrons, the theoretical results deviate from the experimental data by orders of magnitude.

For the example of a Cu metal vapour-or vacuum arc the cathode drop is found to be approximately $U_c = 15$ [V], the electron temperature about $T_- = 25000$ [°K] and the ratio $j_+/j_- = 0.5$.

Since the analysis allows for multiple ionization the presence of multiply charged ions in the spot area can be calculated.

The results of this investigation justify within the E-areas the approximations used in the analysis for the copper arc in an earlier investigation [1]. Outside these E-areas a recalculation with the new results derived here may cause markable changes.

Introduction

The cathode spots of electric arcs are operating under extreme physical conditions which render their analysis difficult. Some of the central quantities like the electron- and the ion current density, the electron temperature, the cathode drop and the thickness of the space charge region are still little understood. Crude estimates have been used motivated by experimental experience or occasionally even by the expectations of the investigator. In many cases some of these quantities were calculated using more or less arbitrary assumptions for the others.

In our previous investigation of "The Stationary Cathode Spot"[1, 2] the above quantities were also subject of limiting assumptions:

Observing that the cathode drop U_c enters into the calculations only ineffectively we treated it as a constant parameter of a numerical value taken from experiment, $U_c = 15$ [V]. With respect to the ion current density j_+ we used Langmuir's saturation current of the plasma calculated under simplifying assumptions. One of them was the limitation to singly ionized ions.

These assumptions made the first approach to the problem tractable. However, I think one can rightly

argue that the experimental evidence for the constancy of U_c and for a numerical value $U_c = 15$ [V] is not conclusive. In particular all points outside the E-area are "purely theoretical" and do not correspond to physical reality, so that the use of experimental information in this area may be misleading.

As to the ionization degree the experimental evidence shows indeed that multiply charged ions originate with high energies from the cathode plasma which suggests that the assumption of single ionization should be reconsidered.

Aim of this Investigation

In this situation the present investigation aims to give a unified theory — a simultaneous and consistent calculation — of the current densities (j_e, j_+), the electron temperature (T_-), the cathode drop (U_c) and the extension of the space charge region (l_{sp}) for given values of the cathode surface temperature (T) and the total current density in the spot (j). Our aim is the calculation of the quantities

$$j_e(T, j) ; \quad j_+(T, j) ; \quad T_-(T, j) ; \quad l_{sp}(T, j) ; \quad U_c(T, j).$$

Of course, the quantities T and j are in principal not available as parameters but determined by the physical laws describing the spot behaviour. However, we can use them as parameters if we neglect two of the governing relations: The energy balance

of the ionization region and the energy balance at the cathode surface. Naturally, these relations are later taken into account as characteristics in the construction of the E-diagram which limits the range of the j and T-values.

The Model

We use the definition of the cathodic discharge part given in [3]: "The cathodic discharge part comprises the whole region of the discharge in which a small displacement of the cathode produces alterations".

As is well known the cathodic electrode part of a high current gas discharge — whether it is a vacuum arc (metal vapour arc) or an arc operating in a residual gas — in most cases shows a structure of multiple spots [4] or cells [5].

Subject of this investigation is one "single spot" or "single cell" of this multiple cathode structure.

In Fig. 1 we present schematically the cathodic discharge component of such a single spot. Specifically this figure shows the "model regions" in which we dissect this cathodic discharge part. We observe the "cathode", the "space charge region", the "inertia limited sheath", the "transition region", the "ionization region or plasma ball" and the "contraction region".

Cathode: The cathode may be considered a homogeneous half space with a plane surface, the temperature in the infinity being T_0, the temperature in the circular spot of the arc onset being uniform and of value T. The physical behaviour of this cathode is determined by the qualities of the material.

Space charge region: The space charge region is governend by space charge effects of the ions moving from the plasma to the cathode, electrons emitted from the cathode and electrons counter diffusing from the plasma ball towards the cathode. The extension of the space charge region is not prescribed but a result of the analysis. In particular we do not accept the common practice to identify the extension of the space charge region with that of the sheath of inertia limited motion. We consider only cases were a one-dimensional description of this region is possible.

Sheath: The sheath is that layer in front of the cathode surface in which the motion of all ions from the ionization region and the electrons from the cathode can be approximated as inertia limited. The sheath region too will be considered as one-dimensional.

Transition region: The transition region is situated in-between the inertia limited sheath on the one side and the ionization region or plasma ball on the other side. It is the range in which the kinetic mechanism changes from collision dominance to inertia dominance. Due to this hybrid situation this region is practically non-understood, and — I am afraid — will be so at the end of this treatment.

Ionization region: The ionization region or the plasma ball is the range of high electron and ion density immediately in front of the cathode surface. It is produced by the energy influx from the strong electric field and particularly from the cathode-emitted electrons entering the plasma with the energy accumulated in the cathode drop of the space charge region (U_c). Both effects cause an electron temperature (T_-) essentially different from the gas temperature (T_0), the latter being assumed identical with the spot temperature of the cathode surface (T). This statement already anticipates that the electrons from the cathode lose their energy preferentially through heating of the plasma electrons and not through direct inelastic collisions, like for instance direct ionizations.

The ionization region lies immediately in front of the cathode and the ionization process (characteristic time $= \lambda^i_- / \bar{v}$; λ^i_- = ionization mean-free-path; $\bar{v}_-$ = average electron velocity) is much faster than the expansion of the plasma cloud (characteristic time $= a \bar{v}$; a = radius of the onset; $\bar{v}$ = average neutral or ion velocity). We therefore expect that the density of the nuclei (neutrals and ions) may be approximated by the neutral density of the equilibrium vapour pressure of the electrode material. On the basis of the same reasoning, we calculate the degree of ionization from the Saha-Equation with an electron temperature different from the gas temperature for a system of a constant number of nuclei.

Finally we emphasize: For our purpose here we do not need anymore knowledge of the ionization region. We want only information about the ion saturation currents flowing to the cathode from the edge of this region. This can be found from the above specifications. Questions of the extension and parameter distributions of this region might be of interest for the detailed calculation of the characteristics, but do not enter the present analysis.

Contraction region: In the contraction region we have the transition from the extreme conditions in the ionization region to those in the column or respectively in the electrode component of the anode. In principal this is an important region. However, as we will see, the quantities which we want to calculate are essentially determined by the system cathode — sheath — ionization region. Just like the details of the ionization region beyond the edge of the sheath, the properties of the contraction region are not relevant.

The Basic Equations

Formulating the laws of the model regions, we bear in mind the application which we intend: We want to calculate cathode drop, current densities, electron temperature and the extension of the space charge region for given values of the surface temperature T and the total current density j. As was already said above we have the choice of T and j only if we neglect two of the governing relations which we have choosen to be the energy conservation law of the ionization region and the energy conservation law at the cathode surface. These two relations — utilizing the results of this paper — will later provide two of the characteristics limiting the existence area in the existence diagram calculations [2, 3, 6, 7].

Consequently we will not engage ourselves here in the analysis of the energy balance of the cathode nor of the ionization region.

Cathode: For the electron emission from the cathode, we use the relations by MURPHY and GOOD [8]

$$j_e = e \int_{\bar{\varepsilon}_m}^{\infty} D(E_c, \bar{\varepsilon}) \, N(\bar{\varepsilon}, T, \varphi) \, d\bar{\varepsilon} \qquad (1)$$

with

$$D(E_c, \bar{\varepsilon}) = \left[1 + \exp\left(-\frac{2i}{\hbar} \int_{x_1}^{x_2} p(x) \, dx\right)\right] \qquad (2)$$

and

$$N(\bar{\varepsilon}, T, \varphi) = (4 \pi m_-/h^3) \, \ln\left\{1 + \exp[-(\bar{\varepsilon} - \varphi)/kT]\right\} \qquad (3)$$

where N, D designate respectively the supply function and the emission probability. φ is the work function, E_c the electric field in the spot at the surface of the cathode, T the surface temperature. e, m_- are respectively the charge and the mass of the electron, $\hbar$ Plancks constant.

The net neutral particle loss at the surface of the cathode is a complicated function of the surface temperature distribution, the geometry of the discharge vessel and the discharge itself.

This very involved problem is not solved yet. It is common practice to neglect the mass loss everywhere, except in the spot itself where one uses the maximum flow density

$$J_{max}^n = p_{ev}(T)/4(m_+ \, kT/3)^{1/2} \qquad (4)$$

with p_{ev} being the loval vapour pressure of the cathode material

$$\log(p_{ev}/[\text{mm Hg}]) = A/T + B \\ + C \cdot \log(T/[^\circ\text{K}]) + DT. \qquad (5)$$

Clearly the true loss obeys the relation

$$J^n \leqq J_{max}^n . \qquad (6)$$

KOGAN and MAKASCHEW [9] have shown that the deviations from the maximum value of Eq. (4) can be relevant.

Sheath and space charge region: According to the model description the electrons emitted from the cathode and the ions flowing from the plasma move inertia limited through the sheath and the space charge region. For most practical cases both regions justify a one-dimensional description.

All space charge effects are taken into account: The ion current from the plasma, the cathodic electron emission current and the counter field diffusion of the electrons from the plasma.

The ion current from the plasma and the electron current from the cathode are easily described by the laws of conservative motion.

The description of the counter diffusing electrons is given by a quasi equilibrium Boltzmann distribution. This requires a voltage drop (U_c), large enough so that the cathode current density of the counter diffusing electrons is but a small fraction of the average electron current fluctuation density at the plasma edge. We can easily see during the analysis that this requirement is always met.

We choose a Cartesian coordinate system with the origin in the boundary separating sheath and transition region (0). The axis points perpendicular to the surface of the cathode into the cathode. The Poisson-Equation reads

$$\frac{d^2 U}{dx^2} = -\frac{e}{\varepsilon_0}(n_+ - n_- - n_e) \qquad (7)$$

where the quantities U, n_+, n_e and n_- designate respectively the electric potential, the number den-

sity of the positive charges, of the electrons emitted from the cathode and of the electrons counter diffusing towards the cathode.

Since ions are neither produced nor destroyed in the sheath, their local density n_+ can be related to the corresponding plasma density near the sheath edge (n_+^0) through the laws of current continuity

$$j_{+\nu} = \text{const} = e\, n_{+\nu}\, \nu\, \bar{v}_+ / 4$$
$$j_+ = \sum_\nu j_{+\nu} = e\, n_-\, \bar{v}_+ / 4 \qquad (8)$$
$$j_+ + j_e = j; \qquad \bar{v}_+ = (3\, k\, T / m_+)^{1/2}$$

and the laws of conservative motion in the form

$$n_{+\mu} = n_{+\nu}^0 (1 - 2\, e\, \nu\, U / k\, T)^{-1/2}. \qquad (9)$$

Here we made use of the above requirement that the counter diffusing electron current density j_{-D} is negligible small. j designates the absolute value of the current density. Further we disposed $U(z=0) = 0$. We therefore find for the total number density of

positive charges

$$n_+ = \sum_\nu \nu\, n_{+\nu} = \sum_\nu (\nu\, n_{+\nu}^0)\, (1 - 2\, e\, \nu\, U / k\, T)^{-1/2}. \qquad (10)$$

Correspondingly we have for the electrons emitted from the cathode

$$n_e = n_{ec}\left[1 - \frac{2\, e\, (U_c - U)}{k\, T} \right]^{-1/2}$$
$$= \frac{|j_e|}{e} \cdot \left(\frac{m_-}{k\, T} \right)^{1/2} \left[1 - \frac{2\, e\, (U_c - U)}{k\, T} \right]^{-1/2}. \qquad (11)$$

Here n_{ec} is the density of the emitted electrons at the cathode surface and U_c the total voltage drop from the sheath edge to the cathode surface.

Finally the Boltzman distribution for the counter diffusing electrons reads

$$n_- = n_-^0 \cdot \exp(e\, U / k\, T_-) \qquad (12)$$

with n_-^0 being the electron density at the sheath edge (excluding emitted electrons) and T_- denoting the electron temperature.

Equations (7) to (12) result in

$$\frac{d^2 U}{dx^2} = -\frac{e}{\varepsilon_0} \left\{ \sum_\nu \frac{\nu\, n_{+\nu}^0}{\left(1 - \frac{2\, e\, \nu\, U}{k\, T} \right)^{1/2}} - \frac{|j_e|}{e} \left(\frac{m_-}{k\, T} \right)^{1/2} \bigg/ \left[1 - \frac{2\, e\, (U_c - U)}{k\, T} \right]^{1/2} - n_-^0 \cdot \exp\{e\, U / k\, T_-\} \right\} \qquad (13)$$

or due to

$$n_-^0 = \sum_\nu \nu\, n_{+\nu}^0 - \frac{|j_e|}{e} \left(\frac{m_-}{k\, T} \right)^{1/2} \bigg/ \left[1 - \frac{2\, e\, U_c}{k\, T} \right]^{1/2} \qquad (14)$$

we may write

$$\frac{d^2 U}{dx^2} = -\frac{e}{\varepsilon_0} \left\{ \sum_\nu \nu\, n_{+\nu}^0 \left[\left(1 - \frac{2\, e\, \nu\, U}{k\, T} \right)^{-1/2} - \exp\{e\, U / k\, T_-\} \right] \right.$$
$$\left. - \frac{|j_e|}{e} \left(\frac{m_-}{k\, T} \right)^{1/2} \left(\left[1 - \frac{2\, e\, (U_c - U)}{k\, T} \right]^{-1/2} - \left[1 - \frac{2\, e\, U_c}{k\, T} \right]^{-1/2} \exp\{e\, U / k\, T_-\} \right) \right\}. \qquad (15)$$

We multiply with (dU/dx) and integrate from the plasma-sheath-edge toward the cathode finding

$$E^2 = \frac{2\, k\, e}{\varepsilon_0} \left\{ \sum_\nu n_{+\nu}^0 \left(T\left[\left(1 - \frac{2\, e\, \nu\, U}{k\, T} \right)^{1/2} - 1 \right] + T_- \cdot \nu\, [\exp\{e\, U / k\, T_-\} - 1] \right) \right.$$
$$+ \frac{|j_e|}{e} \left(\frac{m_-}{k\, T} \right)^{1/2} \left(T\left[\left(1 - \frac{2\, e\, (U_c - U)}{k\, T} \right)^{1/2} - \left(1 - \frac{2\, e\, U_c}{k\, T} \right)^{1/2} \right] \right.$$
$$\left. \left. - T_- \left[1 - \frac{2\, e\, U_c}{k\, T} \right]^{-1/2} \cdot (\exp\{e\, U / k\, T_-\} - 1) \right) \right\} + E_0^2 \qquad (16)$$

where E designates the electric field at x and E_0 that at the sheath edge.

In particular the cathodic field — important for the emission — is given by

$$E_c^2 = \frac{2\, k\, e}{\varepsilon_0} \left\{ \sum_\nu n_{+\nu}^0 \left(T\left[\left(1 - \frac{2\, e\, \nu\, U_c}{k\, T} \right)^{1/2} - 1 \right] + T_- \cdot \nu\, [\exp\{e\, U_c / k\, T_-\} - 1] \right) \right.$$
$$\left. + \frac{|j_e|}{e} \left(\frac{m_-}{k\, T} \right)^{1/2} \left(T\left[1 - \left(1 - \frac{2\, e\, U_c}{k\, T} \right)^{1/2} \right] - T_- \left[1 - \frac{2\, e\, U_c}{k\, T} \right]^{-1/2} (\exp\{e\, U_c / k\, T_-\} - 1) \right) \right\} + E_0^2. \qquad (17)$$

The cathode drop must be found from the relation

$$l_{sh} = \lambda_+^{p'}(T) = \int_{U=0}^{U=U_c} dU/E(U) \qquad (18)$$

with E taken from (16). The quantity $\lambda_+^{p'}(T)$ designates the smallest value

$$\lambda_+^{p'}(T) = \text{Min}(\lambda_{+\nu}^p) \qquad (19)$$

of the mean-free paths for momentum exchange

$$\lambda_{+\nu}^p = \left[n_0 Q_{+\nu 0} + n_-^0 \frac{m_-}{m_+} \frac{4\pi}{9} \left[\frac{e^2 \nu}{4 \varepsilon_0 k T_-} \right]^2 \right. $$
$$\left. \cdot \ln\left(\frac{k^2 T_-^2}{e^4 (n_-^0)^{2/3}} \right) \right]^{-1} \qquad (20)$$

of those kinds of multiply ionized atoms which contribute essentialy to the ion current to the cathode. In Eq. (20) $Q_{+\nu 0}$ characterizes the cross section for the interaction of an ν-times charged ion with a neutral atom. Ion-ion interaction is as usual neglected since momenentum is conserved during the collision. The second term on the right hand side of Eq. (20) accounting for the ion electron interaction is usually very small due to the high persistence factor of the ions caused by the large ion-electron mass ratio.

Equations (15) to (19) provide a complete description of the sheath region if the initial values E_0 and n_+^0 are known. These quantities should be deduced from the analysis of the ionization and transition region.

Ionization and transition region: As was already argued above our interest is not a complete analysis of the ionization and transition region — very likely an unsurmountable task — but an appropriate approximation of the quantities n_+^0 and E_0.

Let us first consider n_+^0. In accord with the basic assumptions for our model, we calculate the number densities n_+^0 from the Saha-Equation

$$\frac{n_{+(\nu+1)} n_-}{n_{+\nu}} = \frac{2 \Sigma_{(\nu+1)}}{\Sigma_{(\nu)}} \left(\frac{2\pi m k T_-}{h^2} \right)^{3/2}$$
$$\cdot \exp\{-e U_{i(\nu+1)}/k T_-\} \qquad (21)$$
$$n_- = \sum_\nu \nu n_{+\nu}; \quad n_0 + \sum_\nu n_{+\nu} = p_{ev}/k T$$

under the assumption that the nuclei number density is constant and given by the vapour density calculated from Equation (5).

As to the quantity E_0 one might naively be tempted to use the field value E in the ionization region calculated from the relation

$$j - j_+ = \sigma E. \qquad (22)$$

Here σ is the plasma conductivity

$$\sigma = (e^2/3 m_- \bar{v}_-) n_- \lambda_-^{p'}(T_-) \qquad (23)$$

with the momentum exchange mean-free path of the electrons

$$\lambda_-^p(T_-) = \left[n_0 Q_{-0} + \right.$$
$$\left. + \sum_\nu n_{+\nu}^0 \nu^2 \cdot \frac{4\pi}{9} \cdot \frac{e^4}{(4\pi \varepsilon_0 k T_-)^2} \ln\left(\frac{k^2 T_-^2}{e^4 n_-^{2/3}} \right) \right]^{-1} \qquad (24)$$

where Q_{-0} describes the cross section for electron-neutral interaction and the electron-electron interaction is neglected due to the momentum conservation in binary collisions.

However, such a choice $E = E_0$ would be very doubtful since according to the wellknown sheath criterion a strong variation of the electric field in the transition region is to be expected.

For the moment let us postpone this problem of the correct determination of the initial field E_0 which obviously is influenced very strongly by the structure of the transition region. In the turn of the evaluation of the sheath qualities, we will find how to deal with this question.

Evaluation Method

Principal procedure and its problems: The solution of the system of Eqs. (1) − (3), (5), (8), (15) − (19), (21) will be carried out on a computer.

On first sight, the problem seems reasonably simple. With T and j being given parameters several ways offer themselves which are practically equivalent.

For instance, if we want to use an iterative procedure, then we start with an estimate of the cathodic field E_c. With T given, we calculate j_e from Equations (1) − (3). j_e yields via Eq. (8) j_+ by subtraction, j_+ known Eqs. (8) and (21) provide the electron temperature T_- and the ion densities $n_{+\nu}^0$. With this knowledge Eqs. (17) and (18) calculate simultaneously the cathode drop U_c and the cathode field E_c which closes the cycle of iteration.

In principal it makes little difference whether we start with E_c or T_- or U_c.

The real problem of the solution reveals itself when one starts to carry out the above scheme: One is obstructed by divergencies and finds that the iteration process does not converge.
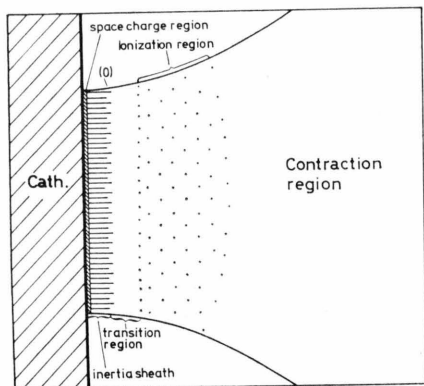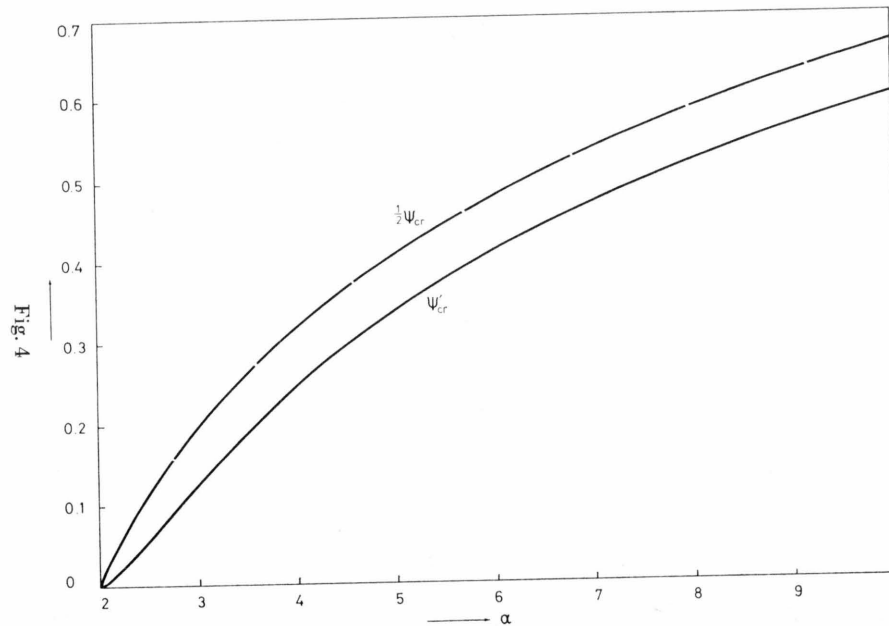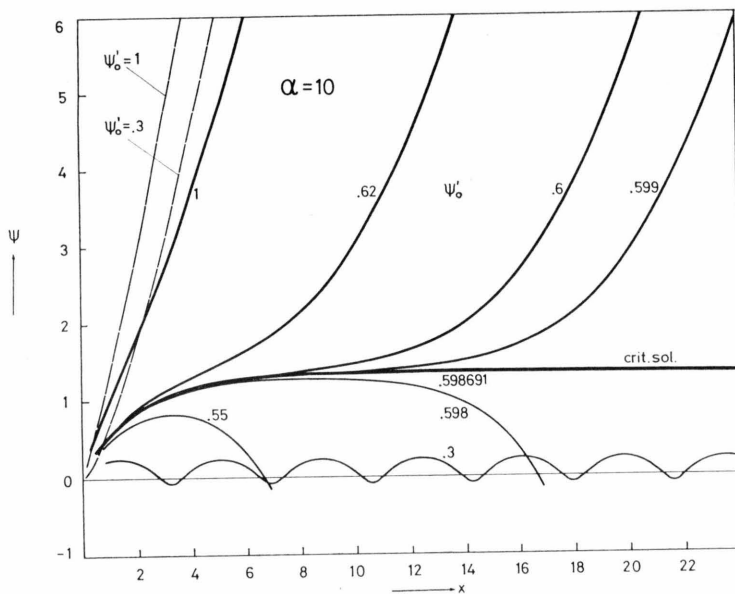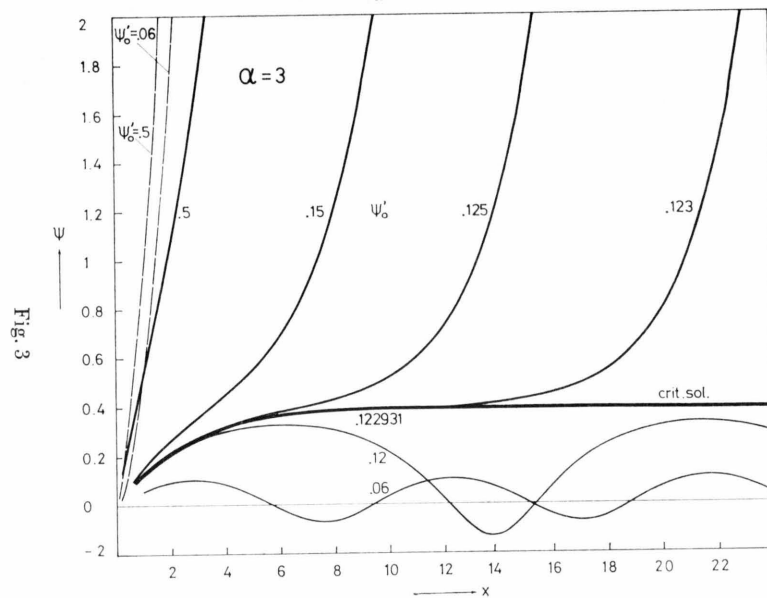
Fig. 1

Fig. 4

Fig. 2

Fig. 3

A detailed analysis of the mathematical background of this phenomenon shows that the sheath edge (0) is a singular point of the differential Equation (15). This fact is vital for the understanding of the mechanism of the cathode spot and we will therefore discuss it separately in the following section.

Singular behaviour of the potential distributions through the sheath; the ineffective boundary condition: The equations quoted above are complicated due to the presence of different species of ions (ν) and the effect of the emission current from the cathode. The phenomenon of singularity does not depend on these two effects. For reason of transparency, we therefore study the phenomenon of singularity in the following for the simpler system of singly ionized ions and a non-emitting cathode.

In this case Eq. (15) simplifies to

$$d^2\psi/dx^2 = (1 + a\,\psi)^{-1/2} - \exp\{-\psi\} \quad (25)$$

where we have used the abbreviations

$$\begin{aligned}
\psi &= - e\,U/k\,T_-\,; \\
x &= z/\lambda_-^D\,; \quad \lambda_-^D = (k\,T_-/4\,\pi\,n_-^0\,e^2)^{1/2}\,; \quad (26) \\
a &= 2\,k\,T_-/m_+\,v_{0+}^2
\end{aligned}$$

with λ_-^D being the Debye length of the plasma electrons.

We use for the initial conditions

$$\begin{aligned}
\psi(x=0) &= \psi_0 = 0\,; \\
d\psi/dx\,|_{x=0} &= \psi_0' \to \text{variabel.} \quad (27)
\end{aligned}$$

The solutions of Eq. (25) for typical parameter values of a and a variable entrance field strength ψ_0' are shown in Figures 2 and 3. For the purpose of comparison, the same figure also shows the well known $U^{3/2}$-law which neglects the contribution of the counter diffusing electrons.

Figures 2 and 3 exhibit all the relevant features: Most important is the fact that there exists a critical entrance field ψ_{cr}' — depending on the parameter

← Fig. 1. Schematic presentation of the model regions of a single cathode spot.

Figs. 2 and 3. Potential distributions through the sheath. For the abbreviations see Equations (26, 27). The solid lines are the solutions with plasma electron counter diffusion. The dotted lines give corresponding solutions calculated with the commonly used $U^{3/2}$-law.

Fig. 4. Asymptotic potential ψ_{cr} and entrance field ψ'_{cr} of the critical solution in the sheath as a function of the parameter a. For the abbreviations see Equations (26, 27).

a — which produces a solution which asymptotically approaches a constant potential value ψ_{cr} for $x \to \infty$. ψ_{cr} is itself a solution of Equation (25).

Entrance field values $\psi_0' < \psi_{cr}'$ yield only solutions which are inconsistent with the model in as far as they imply the existence of trapped ions respectively do not justify the use of the Boltzmann distribution for the counter diffusing electrons.

For entrance fields $\psi_0' > \psi_{cr}'$ the solutions are consistent with the model. For very large entrance fields, they approach the commonly used $U^{3/2}$-solutions. With decreasing $\psi_0' \to \psi_{cr}'$ however, our solutions stay for an increasing distance l_n close to the potential value ψ_{cr} and only then take off from this potential in a way which in essence is similar to the $U^{3/2}$-law. The singular character of our initial value $\psi_0' = \psi_{cr}'$ is reflected in the fact, that the variation dl_n/dx goes to ∞ for infinitesimal decreases in ψ_0'.

Figure 4 shows the critical potential ψ_{cr} and the critical entrance field ψ_{cr}' as a function of the parameter a. We note the monotonous increase of those quantities with a. In particular we see that both quantities approach the value zero at $a = 2$. This latter result reflects the so called "Bohm Sheath Criterion" which in the field of probe theory is generally used neglecting the effects of the initial field.

From the preceeding discussion we draw the following conclusions:

a) The potential distribution within the sheath depends decisively on the counter diffusing electron effect as was already recognized in an earlier paper [10].

b) In general the potential distribution in the sheath may be composed of a part of constant potential ψ_{cr} and a region in front of the cathode where the $U^{3/2}$-law holds. Correspondingly the sheath (l_{sh}) may be composed of a quasi neutral part of extension (l_n) and a space charge region in front of the cathode of thickness ($l_{sh} - l_n$). The extension l_n of the quasi neutral region depends critically on the entrance field strength and the ratio of the electron and ion temperature. It may vary accordingly from zero to infinity.

c) The singular dependence of the sheath solutions on the boundary values (ψ_0') at the sheath entrance renders the boundary condition ineffective. The reason is that the ever present physical fluctuations cause deviations from the given boundary value which admit practically all possible solutions.

Due to this ineffectiveness of the boundary condition an additional criterion has to be found which decides in what mode the discharge operates.

d) The existence of a discrete or continuous set of possible modes due to the ineffectiveness of a certain boundary condition is not new in the field of gaseous electronics. It is also known that in this case the selection of the expected mode should be reached on the basis of stability considerations studying first order perturbations. It is further well known that the corresponding analysis in the field of gas discharges is usually too involved. One therefore generally uses for the selection the "minimum potential principal" which – in some cases – has been traced back to the "minimum entropy production principal" [11–13]. The application of this principal to our problem determines that we will find in the sheath that potential distribution which yields the minimum value of the total potential requirement for the cathodic discharge part.

Mode selection through the minimum potential principal: The sheat has a given extension $l_{sh} = \lambda_+^{p'}$. With the understanding in the previous section, the whole sheath can be subdivided into two parts: One being quasi neutral and of length l_n extends from the transition region towards the cathode. The other being space charge dominated and governed by the $U^{3/2}$-law has the extension $l_{sp} = l_{sh} - l_n$ and lies immediately in front of the cathode.

Due to the ineffectiveness of the boundary condition at the plasma sheath edge (0), all modes belonging to different values of the initial field E_0 are admitted. The variation of E_0 corresponds – as we have seen – to a variation of l_n respectively l_{sp}. We may therefore use l_{sp} to characterize the various modes. We choose l_{sp} as our mode parameter.

The total potential of the spot for a given mode (l_{sp}) can be composed in the form

$$U_T = U_c(l_{sp}) + U_{0I} + U_I(l_{sp}). \quad (28)$$

Where U_c designates the potentil of the space charge region which is given by the $U^{3/2}$-law in the form

$$U_c = \left[\frac{9}{4\,\varepsilon_0} \left(\frac{m_+}{2\,e} \right)^{1/2} j_+ \right]^{2/3} (l_{sp})^{4/3} = c_1 (l_{sp})^{4/3},$$

$$c_1 = \left[(9/4\,\varepsilon_0) (m_+/2\,e)^{1/2} j_+ \right]^{2/3}. \quad (29)$$

U_{0I} designates that part of the potential of the ionization region which is independent, U_I that part which is dependent on the mode parameter l_{sp}.

Since we are looking for the potential minimum as a function of the mode parameter l_{sp} the contribution U_{0I} is not of interest.

The quantity $U_I(l_{sp})$ is mainly determined by the loss of counter diffusing electrons to the cathode. This process causes an additional particle loss not necessary for the current transport. The additional particle loss requires an additional energy supply in the ionization region. Since only the mode parameter (l_{sp}) is changed, all other conditions kept constant, this means we need an additional potential drop U_I in the ionization region.

The corresponding energy balance can be written in the form

$$j_-\,U_I(l_{sp}) \geqq \overline{U}_i\,j_{-D}$$
$$= \overline{U}_i(e\,n\,\bar{v}_-/4)\cdot\exp\{-e\,U_c/k\,T_-\}. \quad (30)$$

Here and in Eqs. (28), (29) U designates absolute values. $\overline{U}_i$ is the average ionization energy of the ions in the plasma.

Using Eq. (23) results in

$$U_I(l_{sp}) = \overline{U}_i(3\,m_-\,\bar{v}_-^2/4\,e\,\lambda_-^p\,E_0)_-^p\cdot\exp\{-e\,U_c/k\,T_-\} \quad (31)$$

or with the local energy balance assumed within the model of the ionization region

$$e\,\lambda_-^p\,E_0 = (f)^{1/2}(m_-\,\bar{v}_-^2/2) \quad (32)$$

in the relation

$$U_I(l_{sp}) = \overline{U}_i[\,(3/2)\,f^{1/2}\,]\cdot\exp\{-e\,U_c/k\,T\} \quad (33)$$

where f is the electron loss factor which is a function increasing with increasing electron temperature starting from the elastic value $f_e = 2.66\,m_-/m_+$.

With that we find for the potential as a function of the mode parameter l_{sp} the relation

$$U_T - U_{0I} = c_1 (l_{sp})^{4/3}$$
$$+ (3\,\overline{U}_i/2\,f^{1/2})\cdot\exp\{-e\,c_1(l_{sp})^{4/3}/k\,T_-\}$$
$$c_1 = [\,(9/4\,\varepsilon_0)\,(m_+/2\,e)^{1/2}\,j_+]^{2/3}. \quad (34)$$

This dependency is shown in Fig. 5 for the example of a Cu-cathode with a current density $j = 10^6\,A/cm^2$ and an electron temperature $T_- = 2 \times 10^4\,°K$. The quantity $f(T_-)$ is not very precisely known. Fortunately it affects our relevant conclusions only little.

The minimum value of the potential U_T can be readily determined from Eq. (34) and is found to occur for the mode parameter

$$(l_{sp})_{min}^{4/3} = \frac{k\,T_-}{e\,c_1}\cdot\ln\left[\frac{e\,\overline{U}_i}{k\,T_-}\cdot\frac{3}{2\,f^{1/2}}\right]. \quad (35)$$
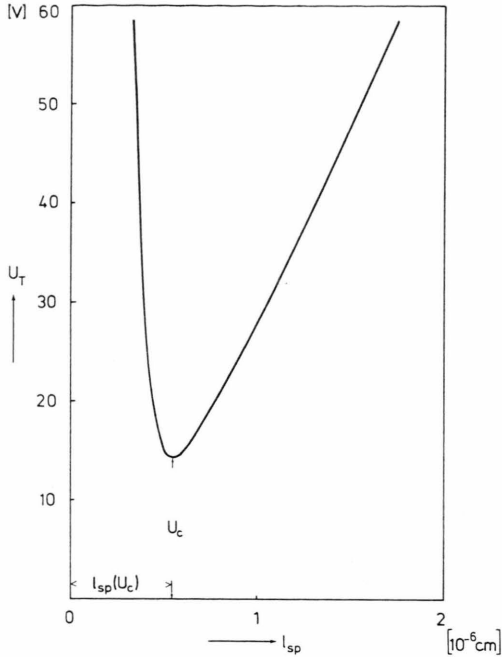
Fig. 5. Total potential U_T as a function of the mode parameter l_{sp} which designates the effective extension of the space charge region.

The corresponding value of the space charge potential U_c of the selected mode is

$$U_c = \frac{kT_-}{e} \ln\left[\frac{e\,\overline{U}_i}{kT_-} \cdot \frac{3}{2\,f^{1/2}}\right]. \qquad (36)$$

Figure 5 demonstrates that the potential rises very rapidly as soon as the parameter l_{sp} decreases below the minimum value $(l_{sp})_{min}$. For this reason, the fact that we neglected the $(>)$-sign in Eq. (30) does not influence our result.

"Renormalized" Sheath Model

The understanding of the previous section justifies the following reformulation of the sheath model which incorporates already the selection of the operating mode: We compose the sheath (l_{sh}) of a quasi neutral part (l_n) and an effective space charge region (l_{sp}).

The description of the quasi neutral range is trivial.

The space charge region will be described by Eqs. (15) and (16) neglecting the contributions caused by the counter diffusing electrons and the initial field which has a negligible influence in that range. We also suppress the term due to the cathodic

electron emission in these equations and take it effectively into account by limiting our calculations to the range

$$j_e/j_+ < (m_+/m_-)^{1/2}. \qquad (37)$$

For a system with singly charged ions this would mean that we discribe the effective space charge region by the $U^{3/2}$-law. With multiple ionization, however, the situation is more complex.

The mode selection in thre previous paragraph requires for the stable operating mode the relation

$$U_c = (kT_-/e)\cdot\ln[3\,e\,\overline{U}_i/2\,kT_-\cdot f^{1/2}(T_-)]. \qquad (38)$$

This equation means no discrepancy to Eq. (18) since we have in our renormalized model an additional unknown quantity l_n or l_{sp}. Equation (18) is just the necessary relation to calculate l_{sp} via

$$l_{sp} = \int_{U=0}^{U_c} dU/E(U). \qquad (39)$$

With this reformulated sheath model, we then have the following *set of equations for the description of the spot system:*

$$j_e = e\int_{\bar{\varepsilon}_m}^{\infty} D(E_c, \bar{\varepsilon})\, N(\bar{\varepsilon}, T, \varphi)\, d\bar{\varepsilon}, \qquad (I.1)$$

$$D(E_c, \bar{\varepsilon}) = \left[1 + \exp\left(-\frac{2\,i}{\hbar}\int_{x_1}^{x_2} p(x)\, dx\right)\right], \qquad (I.2)$$

$$N(\bar{\varepsilon}, T, \varphi) = (4\,\pi\,m_-/h^3)\cdot\ln[1 + \exp\{-(\bar{\varepsilon}-\varphi)/kT\}], \qquad (I.3)$$

$$\frac{n_{+(\nu+1)}\,n_-}{n_{+\nu}} = \qquad\qquad\qquad\qquad (I.4)$$
$$\frac{2\,\Sigma_{(\nu+1)}}{\Sigma_{(\nu)}}\left(\frac{2\,\pi\,m\,kT_-}{h^2}\right)^{3/2}\cdot\exp\{-e\,U_{i(\nu+1)}/kT_-\},$$

$$\log(p_{ev}/[\text{mm Hg}]) = A/T + B + C\cdot\log(T/[^\circ K]) + D\,T, \qquad (I.5)$$

$$j_+ = e\,n_-(3\,kT/16\,m_+)^{1/2}, \qquad (I.6)$$

$$j = j_+ + j_e, \qquad (I.7)$$

$$(dU/dx)^2 = E^2 = (2\,e/\varepsilon_0)\,kT\sum_\nu n_{+\nu}^0(1 - 2\,e\,\nu\,U/kT)^{1/2}, \qquad (I.8)$$

$$E_c^2 = (2\,e/\varepsilon_0)\,kT\sum_\nu n_{+\nu}^0(1 - 2\,e\,\nu\,U_c/kT)^{1/2}, \qquad (I.9)$$

$$U_c = (kT_-/e)\cdot\ln[3\,e\,U_i/2\,kT_-\cdot f^{1/2}(T_-)], \qquad (I.10)$$

$$l_{sp} = (\lambda_+^{p'} - l_n) = \int_{U=0}^{U_c} dU/E(U), \qquad (I.11)$$

$$j_e/j_+ \leq (m_+/m_-)^{1/2}, \qquad (I.12)$$

$$n_- = \sum_\nu \nu\,n_{+\nu}^0, \qquad n_0 + \sum_\nu n_{+\nu} = p_{ev}/kT. \qquad (I.13)$$

Results and Discussion

The results of the numerical computer evaluation of the set of Eq. (I) for our example of the copper vacuum arc is demonstrated in Figures 6 – 8.

We summarize: The *cathode drop* U_c as a function of T and j decreases with decreasing current density j and with increasing temperature T. The variation is small. Considering Fig. 6 one has to realize that most of the (T, j)-points in the diagram are excluded from existence by the energy balance at the cathode and the energy balance of the ionization region which limit the possibility of existence to the areas (0) and (1) shaded in the figure. It is remarkable that within these (0, 1)-modes, the value of the cathode drop U_c is indeed practically constant and close to the value which was used in the calculations [1, 2] with reference to the experimental evidence.

The *electron temperature* T_- demonstrated in Fig. 7 shows a (T, j)-dependency very similar to that of U_c. This seems not surprisinig in the light of Equation (I.10). However, we should bear in mind that the electron temperature via the Saha-
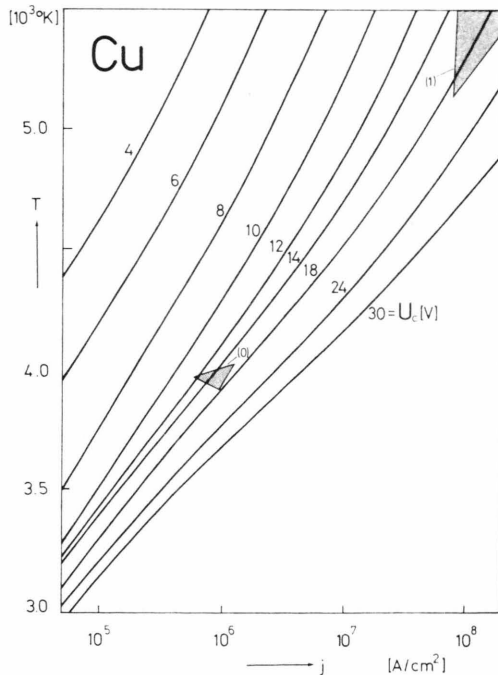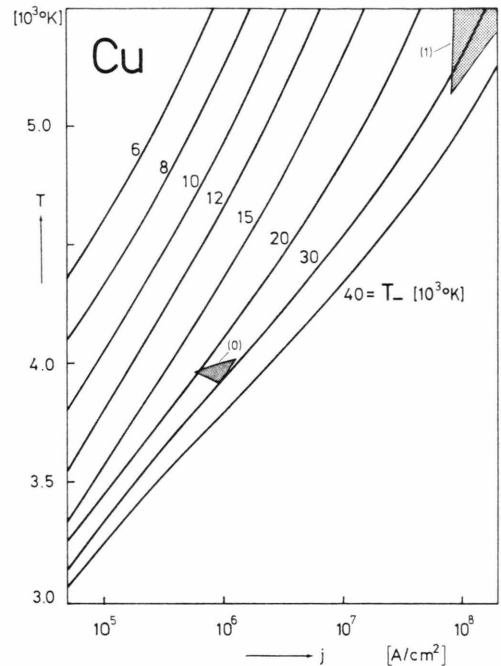

Fig. 7. The average electron temperature T_- is shown for the conditions specified in the caption of Figure 6.

Equation is a critical parameter of the discharge and therefore – even so its variation may appear small in Fig. 7 – has a decisive influence on the physical situation in the ionization region and with that the whole cathode region. In the existence ranges indicated in the figure, the electron temperature is about 25 000 °K.

The electron current density j_e and ion current density j_+ is shown in Figure 8. We recognize that the contribution of the ions to the total current density decreases with increasing curernt density j and increasing spot temperature T. For temperatures of 3500 °K and current densities of 10^5 A/cm^2 the ratio j_+/j may be as high as 0.8. In the range of the existence area of our (0)-mode this ratio is of the order 0.3 and for the (1)-mode it may be smaller than 0.01.

The curve $j_{th}/j_e = 0.1$ which describes the ratio of the thermal electron emission to the total electron emission, proves that in the main part of our (T, j)-diagram – definitely in the range of our existence areas – the current emission from the cathode is dominated completely by field and not by thermal effects.

We also find in the figure three graphs denoted by $n_{++}/n_+ = 0.01$, 0.05, and 1.0. They allow us to

Fig. 6. Cathode drop U_c as a function of the spot temperature T and the current density j for the example of the Cu-vacuum arc. Shaded areas are existence regions for (0) and (1) modes calculated in [1, 2].
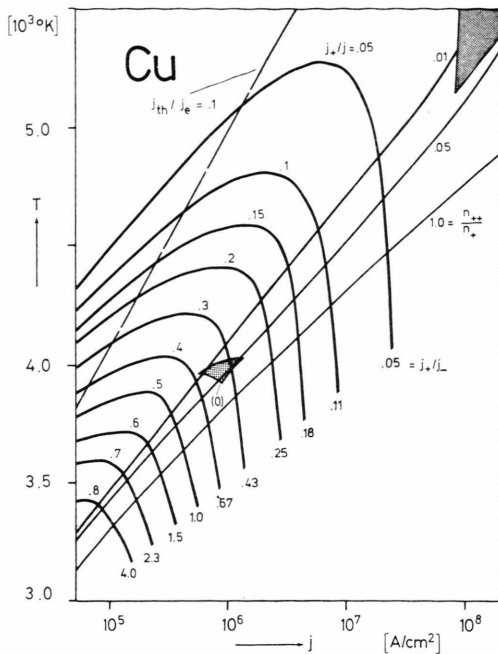
Fig. 8. This figure informs about the current transport in front of the cathode spot for the conditions specified in the caption of Figure 6. The heavy solid lines describe the ion current density fraction j_+/j respectively the ion-electron current density ratio j_+/j_-. The light solid curves indicate constant values of the ratio of doubly ionized and singly ionized ions n_{++}/n_+. Finally to the right of the dotted line the thermal electron emission j_{th} contributes less than ten percent of the total electron emission j_e.

duce approximately the E-areas of the treatment in [1,2]. The shape of the characteristics in other parts of the $(T-j)$-diagram may, however, be seriously affected by the change in the cathode drop and the contribution of the multiply charged ions. It cannot be excluded that such a recalculation utilizing the more advanced understanding of this paper may produce additional E-areas.

Problems still to be solved

The understanding of the mechanism of the sheath and the space charge region communicated in this paper allowed a unified calculation of the quantities U_c, j_e, j_+, T_- and l_{sp} as a function of the parameters T and j.

We stated that a recalculation of the E-diagram of the Cu metal-vapour arc on the basis of the new information would approximately reproduce the E-areas found in the investigations [1,2], but may also result in additional E-areas. Such a recalculation would therefore be of interest.

A recalculation would be valuable also for another reason. In the treatment of [1,2] the evaporation loss from the cathode is described by the maximum value given in Eq. (4) of this paper. A reformulation of this quantity should not only take into account the results of a recent calculation of KOGAN and MAKASCHEW [9] but also the effect which is caused by the drag forces of the ions on the evaporation loss. We expect that such a correction of the evaporation loss could produce a shift of the E-areas published in [1,2], — probably to smaller values of the current density.

An effect on the E-diagram can also be expected from the consideration of a possible roughness factor for the electric field at the cathode surface in the spot area. Such a roughness factor may be essential even if the metal surface is in a molten state in this area.

Finally we point to the fact that according to our present calculation large numbers of doubly, triply and even higher charged ions are present in the plasma ball of the spot. Their densities are of the order

$$n_{+2} = O(10^{17}) ; n_{+3} = O(10^{12}) ; n_{+4} = O(10^7). \quad (40)$$

When such multiply charged ions are transformed at the cathode surface to a singly charged ion their maximum energies from the neutralization and the

judge the effect of multiple ionization. According to these graphs the contribution of doubly charged ions to the total ion density may reach about five percent in our existence areas. Note particularly, how rapidly the contribution of the doubly charged ions increases in the neighbourhood of the existence area when we approach higher current densities j and lower spot temperatures T.

The following *Conclusions* are drawn from these results:

Within the range of the E-areas calculated in the investigations [1,2], the assumption of a constant cathode drop $U_c = 15$ [V] is corroberated by our theoretical results. In other parts of the E-diagram quite different values of U_c are found.

Within the range of the E-areas, the ion current density contribution of multiply charged ions is small, of the order of five percent or less. However, in other parts of the $(T-j)$-diagram it may carry a very substantial part of the total ion current.

Consequently, a recalculation of the E-areas of the Cu metal-vapour arc can be expected to repro-

cathode drop are quite high, namely.

$$\varepsilon_{+\nu} = \sum_{\mu=2}^{\nu} e(U_{i\mu} - \varphi) + e U_c, \qquad (41)$$

$$\varepsilon_{+2} = 34 \text{ [eV]}; \quad \varepsilon_{+3} = 66 \text{ [eV]}; \quad \varepsilon_{+4} = 120 \text{ [eV]}.$$

Even though we have no detailed information about the accommodation coefficients of these processes these energies offer a basis for an explanation of the

surprising phenomenon that ions have been observed emerging from the cathode spot area with energies usually between $20-60$ [eV], i. e. with energies well above the total typical vacuum voltage [14].

Further study of this question, in particular the consideration of the mechanism by which the ions escape the spot area without loosing their energy, is in progress.

[1] G. ECKER, Betr. a. d. Plasmaphysik II, 406 [1971].
[2] G. ECKER, General Electric TIS Report 71-C-195 [1971], Schenectady, New York.
[3] G. ECKER, Erg. exakt. Naturwiss. 33, 1 [1961].
[4] G. ECKER, Z. Physik 136, 556 [1954].
[5] I. G. KESAEV, Cathode Processes in the Mercury Arc, State Power Engineering Press, Moscow 1961.
[6] G. ECKER, Z. Naturforsch. 25 a, 935 [1971].
[7] G. ECKER, Klassische Probleme der Gaselektronik in moderner Sicht, Rheinisch-Westfälische Akademie der Wissenschaften, Vorträge 1972, Westdeutscher Verlag, Opladen.

[8] E. L. MURPHY and R. H. GOOD, JR., Phys. Rev. 102, 1464 [1956].
[9] M. H. KOGAN and H. K. MAKASCHEW, J. Mechanics of Liquids and Gases 6, 3 [1971].
[10] G. ECKER, Z. Physik 135, 105 [1953].
[11] M. STEENBECK, Wiss. Veröffentl. Siemens Concern XIX (1940), 59.
[12] TH. PETERS, Z. Phys. 144, 612 [1956].
[13] M. STEENBECK, Plasma Physik 1, 1 [1960/61].
[14] W. D. DAVIS and H. C. MILLER, J. Appl. Phys. 40, 2212 [1969].

The Non-Stationary Metal Vapour Arc

G. ECKER

Institut für Theoretische Physik, Ruhr-Universität Bochum

This investigation studies the effect which the motion of the cathode spot has on its E-diagram. The motion is depicted as a sequence of steps of a finite residence time.

The spot motion affects essentially only the energy characteristics T_e which in comparison to the stationary characteristics T_{es} are shifted to smaller values. Hereby the critical currents I_0, I_1 are raised in comparison to the corresponding stationary limits I_{0s}, I_{1s}.

Particularly attractive are the phenomena found in connection with the dependence of the spot velocity v on the spot current I. If the spot velocity increases with the spot current stronger than $v \propto I^{1/2}$ then the E-diagram reveals the existence of an upper limit I_u for the spot current. This result can be used to explain qualitatively the experimentally observed phenomena of "spot multiplicity" and "spot extinction".

Quantitative conclusions are obstructed by the lack of knowledge about the velocity dependence on the spot current, $v(I)$. Experimental and theoretical studies to provide a better understanding of the physical background and the analytical laws describing the motion of the cathode spots are urgently needed.

Introduction

In an earlier investigation of the metal vapour arc [1] we considered the stationary cathode spot (SCS). Using the E-diagram technique [2, 3] we studied characteristics of charge conservation, particle conservation and energy conservation. Apart from a relatively close limitation of the current densities and the temperatures in the cathode spot we observed the occurrence of two different spot-modes (0, 1).

We further found a requirement for a minimum

spot current (I_0, I_1) for the stationary operation of a cathodic onset in these modes.

In the consecutive treatment of the metal vapour or vacuum arc [4] we improved the understanding of the cathode spot mechanism by including simultaneously the analysis of the sheath and the space charge region and by taking into account multiply charged ions. In this way we found the variation of the electron temperature T_- and the cathode drop U_c with the spot surface temperature T and the total current density j.

In principal these new results ask for a recalculation of the E-areas derived in [1] utilizing the more accurate data of [4]. This analysis is in progress but